6.3 - Logarithms and Logarithmic Functions

Warmup

1. $\log_7 \sqrt[3]{49}$ $\frac{2}{3}$

2. $\log_3 \sqrt[5]{9}$ $\frac{2}{5}$

3. $\log_{1/2} 8$ -3

4. $\log_{1/3} 27$ -3

5. $\log_2 \sqrt[3]{\frac{1}{4}}$ $-\frac{2}{3}$

6. $\log_{10} \frac{1}{\sqrt{1000}}$ $-\frac{3}{2}$

7. $\log_7 x = 2$ 49

8. $\log_6 x = 3$ 216

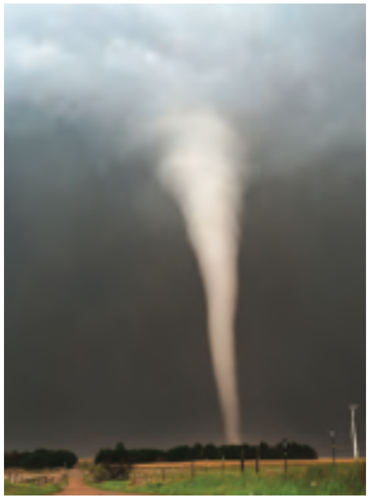
9. $\log_9 x = -\frac{1}{2}$ $\frac{1}{3}$

10. $\log_6 x = 2.5$ $36\sqrt{6}$

11. $\log_4 x = -\frac{3}{2}$ $\frac{1}{8}$

12. $\log_{1/9} x = -\frac{1}{2}$ 3

Chapter 6 Exponential and Logarithmic Functions



1. Exponential Growth and Decay Functions
2. The Natural Base e
3. Logarithms and Logarithmic Functions
4. **Transformations of Exponential and Logarithmic Functions**
5. Properties of Logarithms
6. Solving Exponential and Logarithmic Equations
7. Modeling with Exponential and Logarithmic Functions

6.4 - Transformations of Exponential and Logarithmic Functions

Transformations

$$f(x) = 3^x$$

How do you transform $f(x)$ to obtain $h(x)$?

$a.\ h(x) = 3^{x+2}$

Translate left 2

$b.\ h(x) = 4(3)^x$

vertical stretch 4

6.4 - Transformations of Exponential and Logarithmic Functions

Transformations

$$f(x) = 2^x$$

How do you transform $f(x)$ to obtain $h(x)$?

$a.\ h(x) = 2^x + 3$

Translate up 3

$b.\ h(x) = 2^{3x}$

Horizontal shrink 1/3

$c.\ h(x) = \left(\frac{1}{4}\right)^x + 7$

$= 2^{-2x} + 7$

Reflect y-axis
horz. shrink 1/2
translate up 7

$d.\ h(x) = \left(\frac{\sqrt{2}}{4}\right)^{-x}$

$= 2^{3x/2}$

Horz. shrink 2/3

6.4 - Transformations of Exponential and Logarithmic Functions

Transformations

$$f(x) = e^x$$

How do you transform $f(x)$ to obtain $h(x)$?

$a.\ h(x) = e^{x+2}$

Translate left 2

$b.\ h(x) = -e^{x-4}$

Translate right 4
Reflect across x-axis

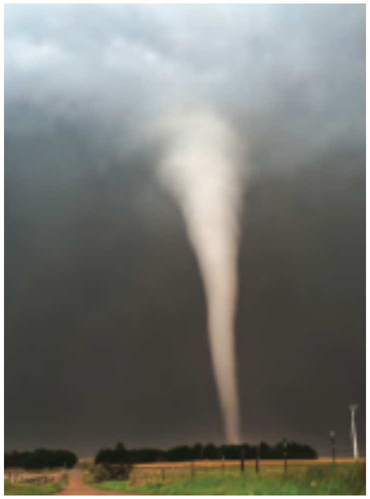
$c.\ h(x) = \left(\dfrac{1}{e^{3x}}\right) + 7$

Reflect y-axis
horz. shrink 1/3
translate up 7

$d.\ h(x) = 2 - \dfrac{1}{e^{x/2}}$

Horz. stretch 2
Reflect y-axis, x-axis
Translate up 2

Chapter 6 Exponential and Logarithmic Functions



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6.5 - Properties of Logarithms

Logarithm Power Property Proof

$$\log_b x^a = y$$

$$x^a = b^y$$

$$\left(x^a\right)^{1/a} = \left(b^y\right)^{1/a}$$

$$x = b^{y/a}$$

$$\log_b x = \frac{y}{a}$$

$$a \log_b x = y$$

6.5 - Properties of Logarithms

Logarithm Power Property

$$\log_3 x^5 = 5\log_3 x \qquad \log_7 5^4 = 4\log_7 5$$

$$\begin{aligned}\log_7\left((xy)^4\right)^z &= z\log_7(xy)^4\\ &= 4z\log_7(xy)\end{aligned}$$

Practice

1. $\log_3 m^4$

   $4\log_3 m$

2. $5\log_2 \dfrac{1}{m^2}$

   $-10\log_2 m$

3. $6\log_4 \dfrac{m}{\sqrt[3]{m^2}}$

   $2\log_4 m$

6.5 - Properties of Logarithms

Logarithm Product Property Proof

$$m = \log_b x \qquad n = \log_b y$$

$$x = b^m \qquad y = b^n$$

$$x \cdot y = b^m b^n$$

$$x \cdot y = b^{m+n}$$

$$\log_b(xy) = m + n$$

$$\log_b(xy) = \log_b x + \log_b y$$

6.5 - Properties of Logarithms

Logarithm Quotient Property Proof

$$m = \log_b x \qquad n = \log_b y$$

$$x = b^m \qquad y = b^n$$

$$\frac{x}{y} = \frac{b^m}{b^n} = b^{m-n}$$

$$\log_b \left(\frac{x}{y}\right) = m - n$$

$$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

6.5 - Properties of Logarithms

Logarithm Product Property - Expanding expression

$$\log_3 xy = \log_3 x + \log_3 y$$

$$\log_7 \frac{x}{y} = \log_7(x \cdot y^{-1})$$
$$= \log_7 x + \log_7 y^{-1}$$
$$= \log_7 x - \log_7 y$$

Practice - Expand

1. $\log_5 xyz$

$\log_5 x + \log_5 y + \log_5 z$

2. $\log_2 \frac{xy}{z^3}$

$\log_2 x + \log_2 y - 3\log_2 z$

3. $\log_4 \left(\frac{x}{y} \cdot (a+b)\right)$

$\log_4 x - \log_4 y + \log_4(a+b)$

6.5 - Properties of Logarithms

Working with exponents

$$5^{2\log_5 x} \quad = 5^{\log_5 x^2} \quad = x^2$$

Convert to base 2

$$\log_{\frac{\sqrt{2}}{2}} x \quad = \log_{2^{-1/2}} x \quad = \log_2 x^{-2}$$

$$2^{\log_{\frac{\sqrt{2}}{2}} x} \quad = 2^{\log_2 x^{-2}} \quad = x^{-2}$$